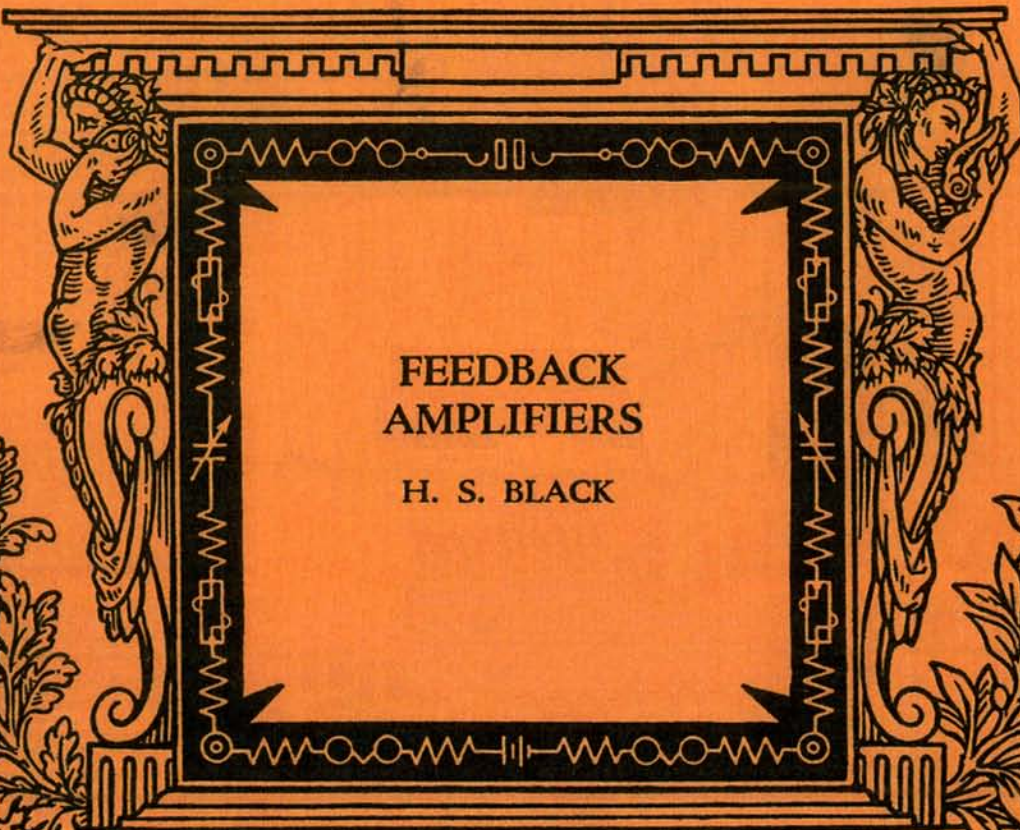


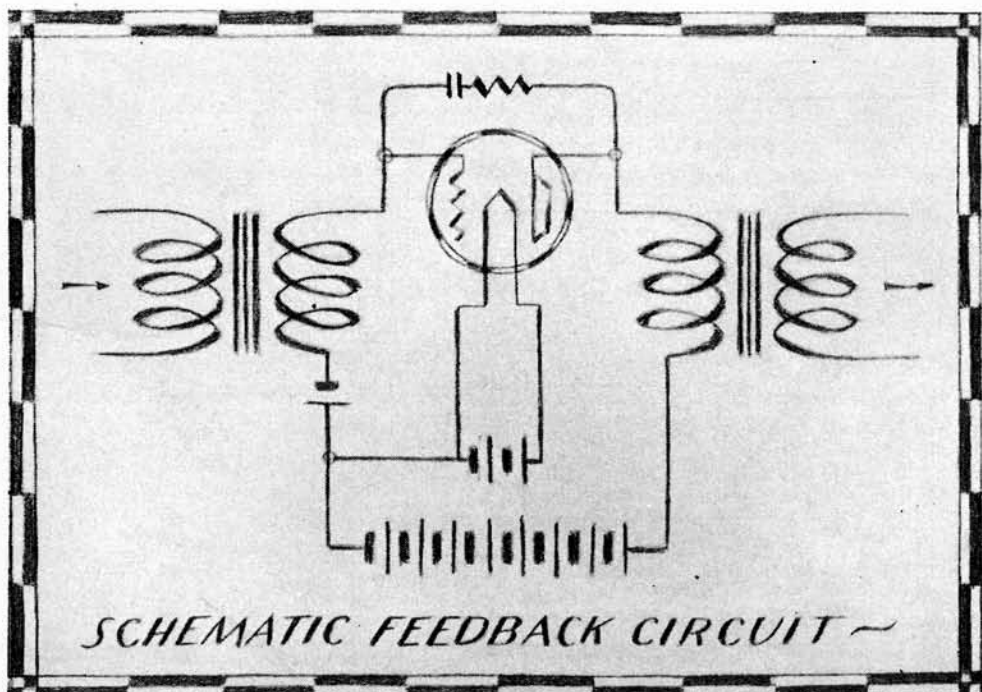
# BELL LABORATORIES RECORD



FEEDBACK  
AMPLIFIERS

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## Feedback Amplifiers

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*Toll Systems Development*

**T**ELEPHONE history is full of dreams come true. Few rosier dreams could be dreamed than that of an amplifier whose overall performance is perfectly constant, and in whose output distortion constitutes only one one-hundred millionth of the total energy, although the component parts may be far from linear in their response and their gain may vary over a considerable range. But the dreamer who awakes in amazement to find that such an amplifier can be built has additional surprises in store for him. These benefits can be obtained by simply throwing away some gain, and by utilizing "feedback action".

A feedback amplifier is one in which a portion of the output is fed back to

the input. The notion of doing this with various types of electrical apparatus has long been familiar; and it has been appreciated that by applying the principle to an amplifier the gain of the device might be increased or decreased, according to the amount and phase of the impulse fed back. It has commonly been supposed, however, that for stable operation there must be a net loss around the closed loop formed by the amplifier and feedback circuits. Furthermore, it has been supposed that in the case where the gain was reduced there was ordinarily no advantage to be had which would justify the loss in amplification; and that, in the case where the gain was appreciably increased, dan-



gerous instability would result, causing at the worst sustained oscillations or "singing" around the closed loop.

It is only recently that further studies revealed the many remarkable and desirable properties that could be secured by properly employing feedback action. Increased constancy of amplification and decreased outputs of noise and harmonics—benefits previously obtainable only by further refinements in the already refined design of vacuum tubes and their power supply circuits—can now be obtained simply and cheaply by designing the amplifier with a feedback circuit whose loss is properly proportioned to the gain of the amplifying circuit. The extent of these improvements is a function of the total gain and phase shift around these two circuits: with a gain of 26 db, noise and the amplitudes of harmonics may be reduced twenty-fold, and constancy or stability of gain may be improved more than twenty-fold. With a total gain of 60 db these improvements can be made a thousand-fold.

There are many ways that feedback action may be produced. In the most general sense feedback action can be defined as the action taking place whenever a resultant vector—representing in this case a voltage or current—is the sum of two vectors one of which is under the control of the resultant. One method of applying a voltage under the control of the resulting voltage is to apply the resulting voltage back upon itself, through a suitable transmission path.

Important and fundamental conditions surrounding the design of feedback amplifiers according to this principle can be derived from some simple considerations embodied in Figure 1. At the top of that figure an amplifier without feedback is shown

by an active or amplifying network with a voltage ratio, output to input, of  $\mu$ . The output voltage,  $E$ , of such an amplifier is  $\mu$  times the input voltage,  $e$ .

The corresponding schematic of a feedback amplifier in Figure 1 shows the active network bridged by a passive feedback circuit, whose output voltage is  $\beta$  times its input voltage. In the closed loop thus formed feedback action takes place, and at the input the effective grid voltage  $e_0$  is no longer equal to  $e$  but becomes the sum of two voltages:  $e$  the applied, and  $\mu\beta e_0$  the feedback voltage. From this equation,  $e_0 = e + \mu\beta e_0$ , the effective input voltage is found to be  $e_0 = e \frac{1}{1-\mu\beta}$ . Thus feedback action alters the applied impulse by the factor  $\frac{1}{1-\mu\beta}$ . Since the output is  $\mu$  times the effective input,  $e_0$ , the output with feedback, is  $\mu e_0 = \frac{\mu e}{1-\mu\beta}$ .

Hence the act of connecting the feedback circuit has divided the principal effect of the applied generator or excitation by the quantity  $(1-\mu\beta)$ . If the magnitude of this quantity is

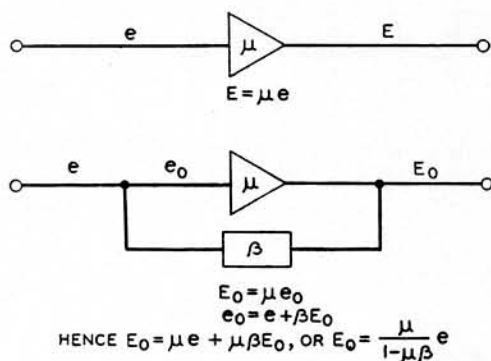


Fig. 1—In an ordinary amplifier (above) the input voltage and the applied voltage have the same value,  $e$ . In a feedback amplifier the input voltage  $e_0$  is the sum of the applied voltage and the voltage fed back. The amplification of the amplifier is thus changed from  $\mu$  to  $\frac{\mu}{1-\mu\beta}$

greater than unity the output is reduced, and the feedback is called "negative feedback"; if this denominator is less than unity, "positive feedback" occurs, increasing the gain of the amplifier.

The more detailed schematics in the headpiece and Figure 2 show two types of feedback: a "voltage" feedback, and a bridge type of feedback. In the second, balanced bridges eliminate the effect of feedback upon the impedances presented by the amplifiers, and remove the influence of the circuits connected to the amplifier upon the amount of feedback.

Since  $\mu$  and  $\beta$  are voltage or current ratios they have both magnitude and phase angle. It is interesting to note that when the magnitude of  $\mu\beta$  is unity and the phase angle zero, the output  $\frac{\mu e}{1-\mu\beta}$  becomes indefinitely large. This is the extreme case of instability, and would, of course, render an amplifier inoperative. In fact it is the one and only necessary condition for self-oscillation. The uniqueness of this condition, however, does not make it easy to avoid. In the the-

oretical case of a perfectly linear amplifier, with infinite power carrying capacity, there are several conditions under which a small disturbance may become exponentially larger with time to an indefinite extent.\* In all practical cases amplifiers are non-linear, and  $\mu$  and  $\beta$  vary with the load. Practically, therefore, whenever a linear amplifier would ultimately give an indefinitely large output, the increasing load will change  $\mu$  and  $\beta$  in the non-linear amplifier until their product is unity with a zero phase angle at a certain frequency, and thus the practical amplifier will sing at this frequency.

For this reason it is far from simple to employ feedback. Very special control is required of phase shifts in the amplifier and feedback circuits, not only throughout the useful frequency band but for a wide range of frequencies above and below it. Unless these relations are maintained singing will occur. Experience has

*\*H. Nyquist has given in the Bell System Technical Journal for January 1932 a simple criterion for designing an amplifier so as to insure freedom from this effect.*

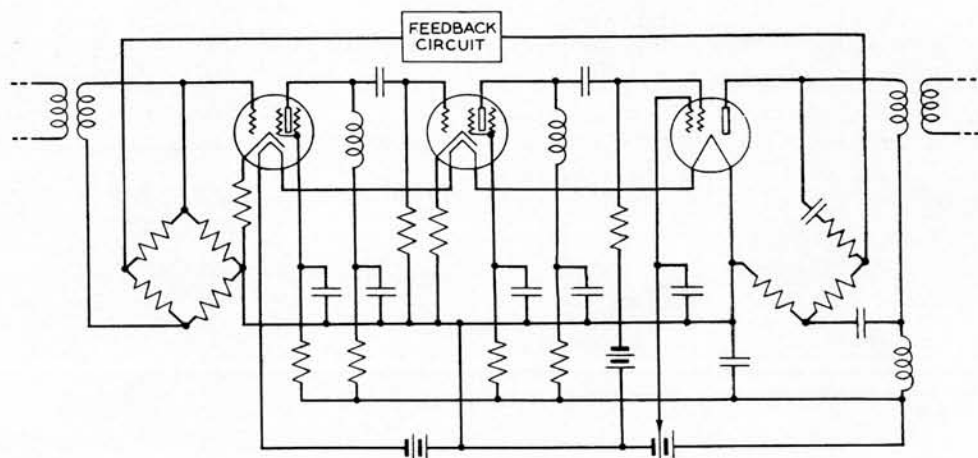


Fig. 2—Although a feedback circuit can sometimes be connected directly between the input and output leads of an amplifier, as shown in the headpiece, it is often more satisfactory to establish the connections through bridges which prevent undesired reactions of the feedback circuit on the input and output circuits

shown, however, that when proper phase relations are provided in the design of an amplifier, its performance is perfectly reliable.

Adding to an amplifier a circuit through which negative feedback will occur is an excellent method of reducing the outputs of noise and harmonics originating in the amplifier. Noise and harmonics may be considered—like the signal input—as generators applied at various points around the closed loop. At each such point, feedback action takes place, and the effective generator is  $\frac{1}{1-\mu\beta}$  times the original. Since noise is independent of the signal, and the harmonics are a function of the signal level, the two must be considered separately.

The signal-to-noise ratio at the output of two similar amplifiers, one employing feedback and the other having the same gain and output level but without feedback, is a convenient measure of the effect of feedback upon the noise. In order that an amplifier with negative feedback have the same gain as the reference amplifier, there must be some extra gain in the  $\mu$ -circuit of the former. If the noise is inserted in the last stage in the same manner and magnitude in the two amplifiers, the feedback amplifier will reduce the applied generator by the factor  $\frac{1}{1-\mu\beta}$ . Inasmuch as the extra gain is not added after the last stage, the signal-to-noise ratio in this case is improved by the factor  $(1-\mu\beta)$ . If the noise is assumed to be introduced in the first stage, the applied noise generator is again changed by the factor  $\frac{1}{1-\mu\beta}$ , but the gain after these two identical first stages is different in the two cases. In the case with feedback the amplification is  $(1-\mu\beta)$  times greater—to compensate for the amount of gain thrown away—

than the reference amplifier which does not have feedback. Hence the signal-to-noise ratio in this case is unchanged. It is apparent from the first case that the power supply and tube design in the last stage of a feedback amplifier may be  $(1-\mu\beta)$  times noisier and meet the same requirements.

Harmonics differ from noise in varying with the signal output. To a first approximation in considering the combined output, of the desired signal and the undesired harmonics, it makes no difference how the distortion was produced: whether by a non-linear condition or by a linear device possessing many extra generators which are the sources of the distortion. In the case without feedback it is assumed that the magnitude of the distortion generator depends only upon the magnitude of the signal output. In many cases it is satisfactory to make the further assumption that this is also true with feedback. Then for a given signal output level the imaginary generators of distortion are fixed in value. When feedback is applied and the input is changed so that the signal output is restored to the value it had without feedback, the “actual” values of these distortion generators will be the same as their original values, but feedback will change their effective values to  $\frac{1}{1-\mu\beta}$  times the original values. With positive feedback, this is an effective increase; with negative feedback, an effective and material decrease. Hence, for the same output, each 10 db decrease in gain at the frequency of the distortion generator will result in reducing by a factor of ten the energy in the distortion.

A closer examination of what happens to the gain of an amplifier when a feedback circuit is added to it, is repaid by the discovery of two in-

teresting types of amplifier. Without feedback the ratio of output to input is  $\mu$ ; with feedback this ratio was shown to be  $A = \frac{\mu}{1-\mu\beta}$ . If the product  $\mu\beta$  is large compared to unity—say 40 or 100 or 1000—then the amplification  $A$  approximates  $-\frac{1}{\beta}$ . With a given frequency characteristic in the  $\beta$ -circuit of such a feedback amplifier, the amplifier as a whole will have the inverse characteristic.

One of the most readily apparent and valuable uses of this sort of amplifier is as an equalizer. Equalization for the frequency distortion occasioned by many types of apparatus is usually accomplished by placing in the direct

path of the signal a network having a frequency characteristic which is the inverse of that to be corrected. For many types of frequency characteristics it is difficult, and for some impossible, to construct a passive network having the exact inverse characteristic. With this type of amplifier, however, it is only necessary to place in the feedback circuit apparatus possessing the same characteristic as that to be corrected. The value of  $\mu\beta$  may be increased to the extent necessary for the desired accuracy of the correction.

There is another method by which the gain can be made exactly equal to  $|\frac{1}{\beta}|$  without resorting to large values of  $\mu\beta$ , as can be seen from the vector diagram in Figure 3. Since  $A$  equals  $\frac{\mu}{1-\mu\beta}$ , vector  $\frac{1}{A}$  equals  $\frac{1-\mu\beta}{\mu}$  and vector  $\frac{1}{\beta A}$  equals  $\frac{1-\mu\beta}{\mu\beta}$  or  $(\frac{1}{\mu\beta} - 1)$ . To construct  $\frac{1}{\beta A}$ , therefore, it is possible to begin with  $\mu\beta$  (Figure 3), having the length  $|\mu\beta|$  and the angle  $\Phi$ , and the unit vector  $1$ . The vector  $\frac{1}{\mu\beta}$  can then be drawn with the length  $\frac{1}{|\mu\beta|}$  and the angle  $-\Phi$ . Finally the vector  $(\frac{1}{\mu\beta} - 1) = \frac{1}{\beta A}$  can be obtained by drawing an arrow from the head of the unit vector to the head of  $\frac{1}{\mu\beta}$  already constructed. These three vectors form triangle  $N$ ; and a line between the heads of  $\mu\beta$  and the unit vector completes another triangle,  $M$ . These triangles are geometrically similar, since the angles  $\Phi$  and  $-\Phi$  are equal in magnitude and the sides including these angles are proportional  $[\frac{1/|\mu\beta|}{1} = \frac{1}{|\mu\beta|}]$ . When triangles  $M$  and  $N$  are made isosceles triangles as shown in Figure 3, the length of  $\frac{1}{\beta A}$  is the same as that of unity, and thus the length of  $A$  is exactly the reciprocal of the length of  $\beta$ . It can be seen from the figure that this condition is satisfied, and the

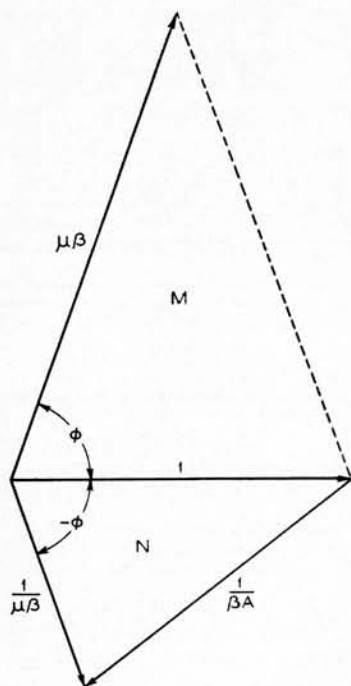


Fig. 3—When  $\mu\beta$  is so chosen that  $M$  is an isosceles triangle (that is, when  $|\mu\beta| = \frac{1}{2} \sec \Phi$ ),  $N$  is also an isosceles triangle and the length of  $\frac{1}{\beta A}$  is the same as the length of  $1$ . Hence the length of  $A$  is the reciprocal of the length of  $\beta$ , and the amplifier has a gain-frequency characteristic the same as the loss-frequency characteristic of the feedback circuit



feedback amplifier will have exactly the same gain as the loss in the  $\beta$ -circuit, whenever  $\frac{1}{2|\mu\beta|} = \cos \Phi$ .

The perfectly constant amplifier, mentioned at the beginning of the

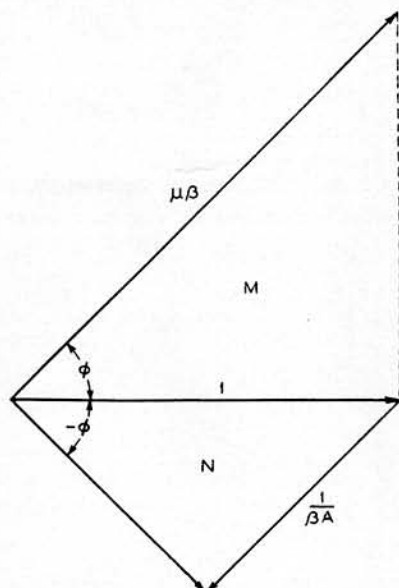


Fig. 4—When  $|\mu\beta| = \sec \Phi$ , M and N are right-angled triangles

article, can be obtained in two ways similar to the two just described for the gain characteristic. It has been shown that when there is a large net gain around the closed loop  $\mu\beta$  becomes large compared to unity, and A the amplification closely approximates  $-\frac{1}{\beta}$ . This means that the effect of  $\mu$  on the overall amplification is less and less. If  $\beta$  is a passive network, constant with time, the effect on the amplification, caused by changing  $\mu$  a small amount  $\Delta\mu$ , is the change  $\Delta\mu$  divided by  $(1 - \mu\beta)$ . With  $\mu\beta$  equal to about 20 db, producing about a 20 db decrease in gain variations in the amplification due to a change of  $\Delta\mu$  will be  $\Delta\mu$  divided by 10. Similarly if the gain were increased 20 db by feedback, the variations due to a

change in  $\mu$  would be multiplied by 10.

In most practical cases it is not the complete vector A that is important, but merely its magnitude, the factor influencing the reading of the voltmeter or current-indicating device in the output. Moreover, it has been found that most amplifier variations are merely variations in the magnitude of  $\mu$ , unaccompanied by appreciable change of phase. To take advantage of these two facts, an amplifier can be designed whose vector diagram is like that in Figures 4 and 5. These diagrams are similar to Figure 3, except that triangles M and N are made right-angled triangles. It can be seen that, for small changes in the magnitude of  $\mu\beta$ , the length of  $\frac{1}{\beta A}$  will be practically unchanged. Furthermore, if there is a slight shift of the right sort in the phase of  $\mu$  at

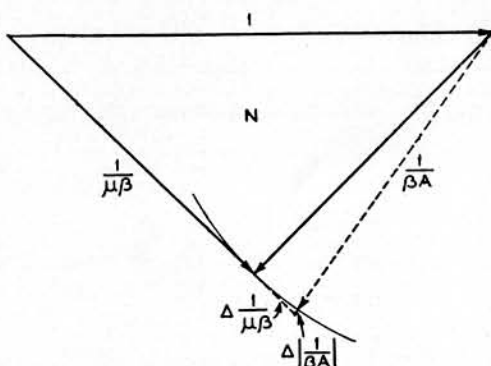


Fig. 5—An enlarged view of triangle N of Figure 4 illustrates the effect on the length of  $\frac{1}{\beta A}$  of small changes in the length of  $\mu$ . A small decrease in the length of  $\mu$  decreases the length of  $\mu\beta$  and hence increases the length of  $\frac{1}{\mu\beta}$  as shown by the dotted extension. The resulting extension of  $\frac{1}{\beta A}$  is only the negligible amount between the circular arc and the line of  $\frac{1}{\mu\beta}$ . Since  $\beta$  is constant, a small change in  $|\frac{1}{\beta A}|$  means a correspondingly small change in  $|A|$ . Hence the gain of the amplifier remains substantially constant despite small changes in gain in the amplifying circuit

the same time that the magnitude changes, a considerable change in the length of  $\frac{1}{\mu\beta}$  will not change the length of  $\frac{1}{\beta\lambda}$  at all.

The triangles M and N will be right-angled whenever  $|\mu\beta| = \sec\Phi$ . It is interesting to notice that this expression holds not only for negative feedback but for positive feedback as well. Hence when  $\frac{1}{1-\mu\beta}$  is greater than unity and  $|\mu\beta| = \sec\Phi$ , the feedback will increase the gain of an amplifier and at the same time render it immune to small variations in  $\mu$ .

The advantages already described and credited to feedback, such as noise reduction, increased linearity, desirable gain frequency characteristics and constancy of gain, are impressive in themselves. Others are also obtainable: improved phase distortion, improved gain load characteristic and better impedances. Their combined practical value is in ena-

bling cheaper apparatus to be used to meet the same requirements, or in meeting requirements that otherwise could not practically be met.

All these apparently paradoxical but extremely valuable results have actually been observed in tests conducted by the Laboratories and many were amply demonstrated in a field trial of feedback repeater-amplifiers at Morristown, N.J. Here 70 feedback amplifiers, designed for use in cable carrier systems, were used in tandem. At full load the energy in the distortion from each amplifier was one ten millionth or less of the signal energy. The maximum change in each amplifier did not exceed .0007 db per volt of change in the plate battery, and at twenty kilocycles the change was only .00005 db per volt. Such constancy as this, hitherto unattainable, will be of the greatest assistance in improving and lowering the cost of carrier telephone systems over long cables.

## FEEDBACK FOR GENERATORS

*The vacuum tubes in telephone repeaters are almost all supplied with filament power from 24-volt storage batteries, "floated" across charging generators. To avoid introducing the ripple produced by these generators into the telephone circuits by way of the filaments, filters are inserted between the batteries and the filament circuits. When the currents flowing between the generator and the batteries are large, however, their fluctuating component may induce the undesired ripple in the transformers associated with the repeaters. By connecting across the charging generator a special amplifier, with its output connected to its input, which amplifies only the alternating components, excluding the direct current, it has been found possible to reduce to a hundredth the power of the ripples in the charging current. This interesting application of the feedback principles, discussed in the foregoing article, was described by F. A. Cowan of the Long Lines Department in ELECTRICAL ENGINEERING for April, p. 590.*



H. S. BLACK came to New York in 1921, after receiving the B. S. degree in electrical engineering from Worcester Polytechnic Institute, and joined the Engineering Department of the Western Electric Company, now these Laboratories. He has since participated in the development of many aspects of carrier telephone systems, including the Type D system, the Type C repeater and line filters, and is the inventor of the type of amplifier, now often called by his name, which he describes in this issue of the RECORD. Mr. Black has in recent years had charge of a group devoted primarily to carrier repeater development.



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